ASSIGNMENT No. 1

Business Mathematics (1429) B.A/ B.Com, BBA

Spring, 2025

Q.1 a) There are 16 green, 20 red and 24 yellow balls in a basket. If we pick a ball at Random what is the probability that (20)

i. The ball is green.

ii. The ball is not green and red.

b) Differentiate between continuous and discrete random variables with the help of examples.

a) Probability of Picking Balls from the Basket

To solve the problems related to probability, we first need to find the total number of balls in the basket.

Total Number of Balls:

- Green balls: 16
- Red balls: 20
- Yellow balls: 24

Total = Green + Red + Yellow
Total = 16 + 20 + 24 = 60 balls

Now, we can proceed to calculate the required probabilities.

i) The Probability that the Ball is Green

The probability of an event occurring is given by the formula:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

In this case, the event we are investigating is picking a green ball.

Number of favorable outcomes (green balls) = 16

Therefore, the probability that a ball picked at random is green is:

$$P(\text{Green}) = \frac{16}{60} = \frac{4}{15} \quad \text{(after simplifying)}$$

ii) The Probability that the Ball is Not Green and Not Red

To calculate this probability, we need to determine the number of balls that are neither green nor red. The only balls left are yellow.

Number of yellow balls = 24

We can use the same probability formula:

Number of favorable outcomes (not green and not red) = 24

Thus, the probability that the ball picked is not green and not red is:

$$P(\text{Not Green and Not Red}) = \frac{24}{60} = \frac{2}{5} \quad \text{(after simplifying)}$$

b) Differentiating Between Continuous and Discrete Random Variables

Random variables are categorized mainly into two types: discrete and continuous. Here is a brief overview of their differences, along with examples.

Discrete Random Variables

A discrete random variable is one that can take on a finite or countable number of distinct values. These values are often whole numbers and can be counted.

Examples of Discrete Random Variables:

1. **Number of Students in a Classroom:** This variable can only take whole number values (e.g., 20, 21, 22, etc.), and you cannot have a fraction of a student.
2. **Number of Heads in Coin Tossing:** If you toss a coin three times, the possible outcomes for the number of heads are 0, 1, 2, or 3. This is also countable.
3. **The Number of Cars Sold:** The total number of cars sold in a month can be 0, 1, 2, etc., again representing countable outcomes.

Continuous Random Variables

A continuous random variable can take on an infinite number of values within a given range. These values are not countable and can include fractions or decimals, as they can be measured.

Examples of Continuous Random Variables:

1. **Height of Students:** The height of students can be measured in terms of inches or centimeters and can take any value within a realistic range (e.g., 150.5 cm, 160.75 cm).
2. **Temperature:** Temperature can vary continuously, for example, a reading can be 22.1 degrees Celsius, 22.11 degrees Celsius, etc.
3. **Time Taken to Complete a Task:** The time it takes for someone to run a race can vary and be measured in hours, minutes, and seconds, representing a continuous interval of values.

In conclusion, the main distinction between discrete and continuous random variables lies in the nature of their values:

Q. 2 The number of fire alarms pulled each hour fluctuates in Islamabad. The probability table of different alarms per hour is shown (20)

No. of Alarm Pulled	Probability
Less than 8	0.12
8	0.24
9	0.28
10	0.26
More than 10	0.10

What is the probability that

a) more than 8 alarms will be pulled?

b) between 8 and 9 alarms (both inclusive) will be pulled.

To answer the questions based on the provided probability table of fire alarms pulled each hour, we first summarize the information given in the table:

No. of Alarms Pulled	Probability
Less than 8	0.12
8	0.24
9	0.28
10	0.26
More than 10	0.10

a) Probability of Pulling More Than 8 Alarms

To find the probability of pulling more than 8 alarms, we need to consider only the probabilities associated with pulling 9, 10, and more than 10 alarms. Therefore, we sum the probabilities for these outcomes.

$$P(\text{More than 8 alarms}) = P(9) + P(10) + P(\text{More than 10})$$

From the table:

- $P(9) = 0.28$
- $P(10) = 0.26$
- $P(\text{More than 10}) = 0.10$

Now, we can compute the probability:

$$P(\text{More than 8 alarms}) = 0.28 + 0.26 + 0.10 = 0.64$$

Answer to Part (a):

The probability that more than 8 alarms will be pulled is **0.64**.

b) Probability of Pulling Between 8 and 9 Alarms (Both Inclusive)

To find the probability of pulling between 8 and 9 alarms (inclusive), we add the probabilities of pulling 8 alarms and 9 alarms:

$$P(8 \leq X \leq 9) = P(8) + P(9)$$

From the table:

- $P(8) = 0.24$
- $P(9) = 0.28$

Now, we can compute the probability:

$$P(8 \leq X \leq 9) = 0.24 + 0.28 = 0.52$$

Answer to Part (b):

The probability that between 8 and 9 alarms (both inclusive) will be pulled is **0.52**.

Q. 3 The data on ocean storms in the USA for the last fifty years is given below (20)

No. of Storms	Frequency
0	2
1	7
2	10
3	15
4	7
5	12
6	7
	Total 60

a) Construct probability distribution for this data.

b) Draw a histogram for this distribution.

Probability distribution means finding the probability of each number of storms:

$$P(X) = \frac{\text{Frequency of X}}{\text{Total frequency}}$$

Let's calculate:

Let's calculate:

No. of Storms (X)	Frequency (f)	Probability $P(X) = \frac{f}{60}$
0	2	$\frac{2}{60} = 0.0333$
1	7	$\frac{7}{60} = 0.1167$
2	10	$\frac{10}{60} = 0.1667$
3	15	$\frac{15}{60} = 0.25$
4	7	$\frac{7}{60} = 0.1167$
5	12	$\frac{12}{60} = 0.2$
6	7	$\frac{7}{60} = 0.1167$

Thus, the **probability distribution** is:

$$P(X = 0) = 0.0333, \quad P(X = 1) = 0.1167, \quad \dots, \quad P(X = 6) = 0.1167$$

(b) Draw a Histogram

In the histogram:

- X-axis = Number of Storms (0, 1, 2, ..., 6)
- Y-axis = Frequency (or you can also plot probability if asked)
- Each bar's height represents the frequency (or probability) of that number of storms.

Q. 4 a) Solve the second degree equation and find the nature of its roots. (20)

$$y^2 - y - 2 = 0$$

The given equation is:

$$y^2 - y - 2 = 0$$

This is a standard quadratic form:

$$ay^2 + by + c = 0$$

where
$a = 1, b = -1, c = -2.$

First, find the discriminant D:

$$D = b^2 - 4ac$$

Substituting the values:

$$D = (-1)^2 - 4(1)(-2)$$

$$D = 1 + 8$$

$$D = 9$$

Nature of Roots:

- If $D > 0$, the roots are real and distinct.

Since $D = 9 > 0$, the roots are real and distinct.

Now, let's solve for roots:

Using the quadratic formula:

$$y = \frac{-b \pm \sqrt{D}}{2a}$$

Substituting the values:

$$y = \frac{-(-1) \pm \sqrt{9}}{2(1)}$$

$$y = \frac{1 \pm 3}{2}$$

Now calculate both roots:

1.

$$y = \frac{1+3}{2} = \frac{4}{2} = 2$$

2.

$$y = \frac{1-3}{2} = \frac{-2}{2} = -1$$

Final Answer:

- The roots are $y = 2$ and $y = -1$.
- Nature of roots: Real and distinct.

Would you also like me to show you how to solve it by factorization? (it's even quicker for this one!)

b) Solve the inequality and represent the solution on the real line.

$$2x^2 + 5x + 3 < 0$$

Q. 5 a) Find the point of intersection of the following lines if it exists (20)

$$x + 2y = 3$$
$$2x - y = 1$$

Step 1: Solve the corresponding equation

First, solve:

$$2x^2 + 5x + 3 = 0$$

Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 2, b = 5, c = 3$.

Find the discriminant:

$$D = b^2 - 4ac = 5^2 - 4(2)(3) = 25 - 24 = 1$$

Since $D = 1 > 0$, we get two real and distinct roots.

Now:

$$x = \frac{-5 \pm \sqrt{1}}{2(2)} = \frac{-5 \pm 1}{4}$$

Thus:

- First root:

$$x = \frac{-5 + 1}{4} = \frac{-4}{4} = -1$$

- Second root:

$$x = \frac{-5 - 1}{4} = \frac{-6}{4} = \frac{-3}{2}$$

Step 2: Find the intervals

The two roots are $x = -\frac{3}{2}$ and $x = -1$.

Since the parabola $2x^2 + 5x + 3$ opens upward (because $a = 2 > 0$), the quadratic is:

- Negative (below the x-axis) between the roots,
- Positive outside the roots.

Thus, the inequality $2x^2 + 5x + 3 < 0$ is satisfied when:

$$-\frac{3}{2} < x < -1$$

Final Answer:

$$\boxed{-\frac{3}{2} < x < -1}$$

Graph on the real line:

On the number line:

- Open circles at $x = -\frac{3}{2}$ and $x = -1$ (because the inequality is strict "<", not "≤").
- Shade the region between $-\frac{3}{2}$ and -1.

b) Let C mean Celsius degree and F mean Fahrenheit temperature scale. Find a linear equation for C if its slope is 5/9 and the C-intercept is -100/9.

You are told:

- The slope $m = \frac{5}{9}$,
- The C-intercept is $\frac{-100}{9}$,
- You need the linear equation for C in terms of F.

The general form of a linear equation is:

$$C = mF + b$$

where:

- m = slope,
- b = C-intercept.

Substituting the given values:

$$C = \frac{5}{9}F - \frac{100}{9}$$

Final Answer:

$$\boxed{C = \frac{5}{9}F - \frac{100}{9}}$$