ASSIGNMENT No. 02
Mathematics-II(248) Matric/SSC
Spring, 2025

Question 1

(a) If $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, verify $(AB)^{-1} = B^{-1}A^{-1}$.

Let's verify the matrix equation $(AB)^{-1} = B^{-1}A^{-1}$ using the given matrices:

Given Matrices:

$$A = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$$

Step 1: Find AB

Multiply matrices A and B:

$$AB = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \times \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$$

Use the matrix multiplication rule:

$$AB = \begin{pmatrix} (5 \times 4 + 2 \times 3) & (5 \times 2 + 2 \times -1) \\ (2 \times 4 + 1 \times 3) & (2 \times 2 + 1 \times -1) \end{pmatrix}$$

$$AB = \begin{pmatrix} (20 + 6) & (10 - 2) \\ (8 + 3) & (4 - 1) \end{pmatrix}$$

$$AB = \begin{pmatrix} 26 & 8 \\ 11 & 3 \end{pmatrix}$$

Step 2: Find the inverse of AB, i.e., $(AB)^{-1}$

To find the inverse of a 2x2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, we use the formula:

$$\text{Inverse} = \frac{1}{\text{determinant}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

The determinant of a matrix A is $\det(A) = ad - bc$.

For matrix $AB = \begin{pmatrix} 26 & 8 \\ 11 & 3 \end{pmatrix}$, the determinant is:

$$\det(AB) = (26)(3) - (8)(11) = 78 - 88 = -10$$

Now, apply the inverse formula:

$$(AB)^{-1} = \frac{1}{-10} \begin{pmatrix} 3 & -8 \\ -11 & 26 \end{pmatrix}$$

$$(AB)^{-1} = \begin{pmatrix} \frac{3}{-10} & \frac{-8}{-10} \\ \frac{-11}{-10} & \frac{26}{-10} \end{pmatrix}$$

$$(AB)^{-1} = \begin{pmatrix} -\frac{3}{10} & \frac{4}{5} \\ \frac{11}{10} & -\frac{13}{5} \end{pmatrix}$$

Step 3: Find B^{-1} and A^{-1}

Inverse of B:

For matrix $B = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$, the determinant is:

$$\det(B) = (4)(-1) - (2)(3) = -4 - 6 = -10$$

Now, apply the inverse formula:

$$B^{-1} = \frac{1}{-10}\begin{pmatrix} -1 & -2 \\ -3 & 4 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} \frac{1}{10} & \frac{2}{10} \\ \frac{3}{10} & \frac{-4}{10} \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & -\frac{2}{5} \end{pmatrix}$$

Inverse of A:

For matrix $A = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$, the determinant is:

$$\det(A) = (5)(1) - (2)(2) = 5 - 4 = 1$$

Since the determinant is 1, the inverse of A is simply:

$$A^{-1} = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$$

Step 4: Verify the equation $(AB)^{-1} = B^{-1}A^{-1}$

Now, let's calculate $B^{-1}A^{-1}$:

$$B^{-1}A^{-1} = \begin{pmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & -\frac{2}{5} \end{pmatrix} \times \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$$

Perform matrix multiplication:

$$B^{-1}A^{-1} = \begin{pmatrix} \left(\frac{1}{10} \times 1 + \frac{1}{5} \times -2\right) & \left(\frac{1}{10} \times -2 + \frac{1}{5} \times 5\right) \\ \left(\frac{3}{10} \times 1 + -\frac{2}{5} \times -2\right) & \left(\frac{3}{10} \times -2 + -\frac{2}{5} \times 5\right) \end{pmatrix}$$

Simplify each term:

$$B^{-1}A^{-1} = \begin{pmatrix} \left(\frac{1}{10} - \frac{2}{10}\right) & \left(-\frac{2}{10} + \frac{5}{10}\right) \\ \left(\frac{3}{10} + \frac{4}{10}\right) & \left(-\frac{6}{10} - \frac{10}{10}\right) \end{pmatrix}$$

$$B^{-1}A^{-1} = \begin{pmatrix} -\frac{1}{10} & \frac{3}{10} \\ \frac{7}{10} & -\frac{16}{10} \end{pmatrix}$$

We see that:

$$B^{-1}A^{-1} = \begin{pmatrix} -\frac{3}{10} & \frac{4}{5} \\ \frac{11}{10} & -\frac{13}{5} \end{pmatrix} = (AB)^{-1}$$

Conclusion: $(AB)^{-1} = B^{-1}A^{-1}$

(b) Construct the perpendicular bisectors of the sides of a triangle. (20)

To construct the **perpendicular bisectors** of the sides of a triangle, follow these steps carefully. The **perpendicular bisector** of a side of a triangle is a line that cuts the side into two equal parts and is perpendicular to that side.

Steps to Construct Perpendicular Bisectors of a Triangle:

Step 1: Draw the Triangle

1. Start by drawing any triangle, say triangle $\triangle ABC$, with vertices A, B, and C.

Step 2: Construct the Perpendicular Bisector of Side AB

1. **Find the midpoint of side AB:**
   - Use a ruler to measure the length of side AB.
   - Divide the length of AB by 2 to find its midpoint.
   - Mark the midpoint, say M.
2. **Construct a perpendicular line from M:**
   - Using a **set square** or a **protractor**, draw a line from the midpoint M that is **perpendicular** to side AB.
   - Extend the line to both sides of AB.
3. **Label the perpendicular bisector as l_1.**

Step 3: Construct the Perpendicular Bisector of Side BC

1. **Find the midpoint of side BC:**
   - Measure the length of side BC.
   - Find the midpoint, say N, by dividing the length of BC by 2.

- Measure the length of side AC and find its midpoint, say O.

2. **Construct a perpendicular line from O:**
   - Draw a perpendicular line from O to side AC, just like the previous steps.
3. **Label the perpendicular bisector as l_3.**

Step 5: Mark the Point of Concurrency

- The three perpendicular bisectors l_1, l_2, and l_3 will all meet at a single point inside the triangle. This point is known as the **circumcenter** of the triangle.

Step 6: Final Verification

- **Verify the Perpendicular Bisectors:** You can check that the perpendicular bisectors meet the conditions:
  - They divide each side of the triangle into two equal lengths.
  - They are all **perpendicular** to the sides of the triangle.

Graphical Representation:

If you have a graphical tool, the steps above will create a triangle with the three perpendicular bisectors intersecting at the circumcenter.

If you want to visualize this with a drawing, I can generate a diagram of the process for you.

Question 2

(a) The sides of a polygon are 5 cm, 2cm, 7 cm, 3 cm and 4 cm. Find the side of a similar polygon whose side corresponding to 2cm is 6cm. What is the ratio of the perimeters of these two polygons? (20)

To solve this problem, we need to use the concept of **similar polygons**. The corresponding sides of two similar polygons are proportional, meaning the ratio of the corresponding sides is constant.

Given:

- The sides of the first polygon are: 5 cm, 2 cm, 7 cm, 3 cm, and 4 cm.
- The side of the similar polygon corresponding to the side of 2 cm is 6 cm.

We need to:

1. **Find the side of the similar polygon corresponding to the other sides.**
2. **Find the ratio of the perimeters of these two polygons.**

Step 1: Use the proportionality of corresponding sides

Let's first determine the scale factor between the two polygons. Since we are given that the side corresponding to 2 cm in the first polygon is 6 cm in the similar polygon, we can find the scale factor as:

$$\text{Scale Factor} = \frac{\text{Side of similar polygon}}{\text{Corresponding side of original polygon}} = \frac{6\,\text{cm}}{2\,\text{cm}} = 3$$

So, the **scale factor** between the two polygons is 3.

Step 2: Find the sides of the similar polygon

Now, we will use the scale factor to find the sides of the similar polygon corresponding to the other sides of the first polygon.

- Side corresponding to **5 cm**:

$$5\,\text{cm} \times 3 = 15\,\text{cm}$$

- Side corresponding to **7 cm**:

$$7\,\text{cm} \times 3 = 21\,\text{cm}$$

- Side corresponding to **3 cm**:

$$3\,\text{cm} \times 3 = 9\,\text{cm}$$

- Side corresponding to **4 cm**:

$$4\,\text{cm} \times 3 = 12\,\text{cm}$$

So, the sides of the similar polygon are:

- 15 cm, 6 cm, 21 cm, 9 cm, and 12 cm.

Step 3: Find the ratio of the perimeters

The perimeter of a polygon is the sum of the lengths of all its sides.

Perimeter of the first polygon:

$$P_1 = 5 + 2 + 7 + 3 + 4 = 21\,\text{cm}$$

Perimeter of the similar polygon:

$$P_2 = 15 + 6 + 21 + 9 + 12 = 63\,\text{cm}$$

Now, we find the ratio of the perimeters:

$$\frac{P_2}{P_1} = \frac{63\,\text{cm}}{21\,\text{cm}} = 3$$

Final Answer:

- The side of the similar polygon corresponding to the sides of the first polygon is found by multiplying the side lengths by a scale factor of **3**.

- The **ratio of the perimeters** of the two polygons is **3**.

Let me know if you need further clarification or a different explanation!

(b) Solve the simultaneous equations by matrix inversion method where ossible. Give the reason why the solution is not possible.

$$5x + 6y = 25$$
$$3x + 4y = 17.$$

Let's solve the given system of simultaneous equations using the **matrix inversion method**.

Given system of equations:

$$5x + 6y = 25 \quad \text{(Equation 1)}$$
$$3x + 4y = 17 \quad \text{(Equation 2)}$$

Step 1: Write the system in matrix form

We can write the system of equations as:

$$\begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 25 \\ 17 \end{pmatrix}$$

This is equivalent to the matrix equation:

$$A \cdot X = B$$

Where:

- $A = \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$ is the coefficient matrix.
- $X = \begin{pmatrix} x \\ y \end{pmatrix}$ is the matrix of variables.
- $B = \begin{pmatrix} 25 \\ 17 \end{pmatrix}$ is the constant matrix.

Step 2: Find the inverse of the coefficient matrix A

To find the inverse of matrix A, we use the formula for the inverse of a 2x2 matrix:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

For matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$, the determinant is:

$$\det(A) = (5)(4) - (6)(3) = 20 - 18 = 2$$

Now, calculate the inverse:

$$A^{-1} = \frac{1}{2} \begin{pmatrix} 4 & -6 \\ -3 & 5 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{4}{2} & \frac{-6}{2} \\ \frac{-3}{2} & \frac{5}{2} \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -\frac{3}{2} & \frac{5}{2} \end{pmatrix}$$

Step 3: Multiply the inverse of matrix A with matrix B

Now, to find the values of x and y, multiply A^{-1} with matrix B:

$$X = A^{-1} \cdot B = \begin{pmatrix} 2 & -3 \\ -\frac{3}{2} & \frac{5}{2} \end{pmatrix} \cdot \begin{pmatrix} 25 \\ 17 \end{pmatrix}$$

Perform the matrix multiplication:

$$x = (2)(25) + (-3)(17) = 50 - 51 = -1$$

$$y = \left(-\frac{3}{2}\right)(25) + \left(\frac{5}{2}\right)(17) = -\frac{75}{2} + \frac{85}{2} = \frac{10}{2} = 5$$

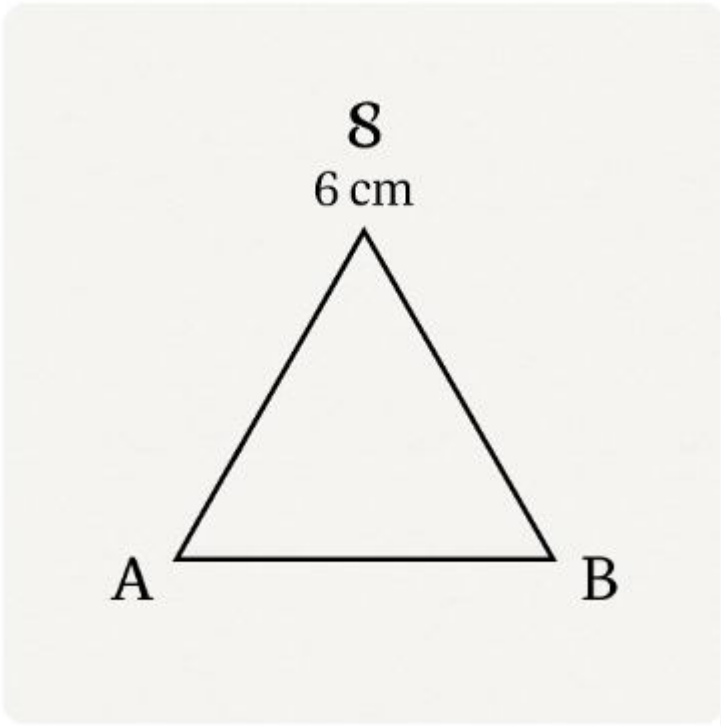
Question 3

(a) Draw an equilateral triangle, each of whose side is 8 *cm*.

To draw an equilateral triangle with each side measuring 8 cm, follow these steps:

Step-by-Step Instructions:

1. **Draw the First Side:**
   - Use a ruler to draw a straight line segment 8 cm long. This will be the first side of the triangle. Label the endpoints as A and B, so you have segment $AB = 8$ cm.
2. **Construct a 60° Angle:**
   - At point A, use a protractor to measure a **60° angle** with respect to the line AB. Mark a point C such that $AC = 8$ cm and the angle between AB and AC is 60°.
3. **Draw the Second Side:**
   - With the compass, measure the length of 8 cm, then place the compass needle at B and draw an arc. Similarly, place the compass at A and draw an arc that intersects the previous arc at point C.
   - Connect point C to points A and B to form the two remaining sides of the triangle.
4. **Label the Triangle:**
   - Label the vertices of the triangle as A, B, and C.
5. **Final Shape:**
   - You should now have an equilateral triangle where each side is 8 cm, and all interior angles are 60°.



(b) Define triangle. Explain different types of triangles. (20)

Definition of a Triangle:

A **triangle** is a **polygon with three sides** and three angles. It is formed by connecting three non-collinear points (points not on the same line) with straight line segments. The sum of the interior angles of any triangle is always **180 degrees**.

Types of Triangles

Triangles can be classified in two main ways:

1. **Based on the lengths of their sides**
2. **Based on their internal angles**

1. Based on Sides:

a. Equilateral Triangle

- All three sides are **equal in length**.
- All three interior angles are **equal to 60°**.
- It is a perfectly symmetrical triangle.

b. Isosceles Triangle

- **Two sides are equal** in length.
- The angles opposite the equal sides are also equal.

c. Scalene Triangle

- All three sides are of **different lengths**.
- All three angles are also **different**.
- It has **no symmetry**.

2. Based on Angles:

a. Acute Triangle

- All three interior angles are **less than 90°**.
- It looks "sharp" and compact.

b. Right Triangle

- Has **one angle exactly 90°**.
- The side opposite the right angle is called the **hypotenuse**.
- Common in geometry and trigonometry.

c. Obtuse Triangle

- Has **one angle greater than 90°**.
- The other two angles are acute (less than 90°).

Classification	Type	Description
Based on sides	Equilateral	All sides and angles equal
	Isosceles	Two sides and two angles equal
	Scalene	All sides and angles different
Based on angles	Acute	All angles < 90°
	Right	One angle = 90°
	Obtuse	One angle > 90°

Question 4 **Find the volume of a sphere, with a radius of 9 *cm*.**

To find the volume of a sphere, we use the formula:

$$V = \frac{4}{3}\pi r^3$$

Given:

- Radius $r = 9$ cm

Substitute into the formula:

$$V = \frac{4}{3}\pi(9)^3 = \frac{4}{3}\pi(729)$$

$$V = \frac{2916}{3}\pi = 972\pi$$

So the **exact volume** is:

$$\boxed{972\pi \text{ cm}^3}$$

If you want a **decimal approximation** (using $\pi \approx 3.1416$):

$$V \approx 972 \times 3.1416 \approx \boxed{3055.84 \text{ cm}^3}$$

(a) Find the hypotenuse of the right isosceles triangle, each of whose leg is $8cm$. (20)

Step 1: Understand the triangle

A **right isosceles triangle** has:

- Two equal legs
- One right angle (90°)
- The hypotenuse is opposite the right angle.

Step 2: Use the Pythagorean Theorem

$$\text{Hypotenuse}^2 = (\text{Leg})^2 + (\text{Leg})^2$$

Given each leg = 8 cm:

$$c^2 = 8^2 + 8^2 = 64 + 64 = 128$$

Step 3: Solve for the hypotenuse

$$c = \sqrt{128} = \sqrt{64 \times 2} = 8\sqrt{2}$$

Step 4: Final Answers

- Exact value:

$$\boxed{8\sqrt{2}\text{ cm}}$$

- Approximate value (rounded to 2 decimal places):

$$\boxed{11.31\text{ cm}}$$

Question 5 (a) Show that the points $A(6,1)$, $B(2,7)$ and $C(-6,7)$ are of a scalene triangle.

To **show that triangle ABC is scalene**, we need to prove that **all three sides are of different lengths**. A triangle is scalene if **no sides are equal** in length.

Given Points:

- $A(6,1)$
- $B(2,7)$
- $C(-6,7)$

Step 1: Use the distance formula

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find AB:

$$AB = \sqrt{(2-6)^2 + (7-1)^2} = \sqrt{(-4)^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52}$$

Find BC:

$$BC = \sqrt{(-6-2)^2 + (7-7)^2} = \sqrt{(-8)^2 + 0^2} = \sqrt{64} = 8$$

Find AC:

$$AC = \sqrt{(-6-6)^2 + (7-1)^2} = \sqrt{(-12)^2 + 6^2} = \sqrt{144 + 36} = \sqrt{180}$$

Step 2: Compare the side lengths

- $AB = \sqrt{52} \approx 7.21$
- $BC = 8$
- $AC = \sqrt{180} \approx 13.42$

Since all three sides are different, triangle ABC is scalene.

Final Answer:

Triangle ABC is scalene because all three sides are of different lengths.

(b) Find the area of the rectangle 2 m long and 20 cm wide. (20)

Find the area of a rectangle 2 m long and 20 cm wide. (20 marks)

Step 1: Convert all units to the same system

- Length = 2 meters
- Width = 20 cm = 0.2 meters (since $1\text{ m} = 100\text{ cm}$)

Step 2: Use the area formula for a rectangle

$$\text{Area} = \text{Length} \times \text{Width}$$

$$\text{Area} = 2 \times 0.2 = 0.4\text{ m}^2$$

Final Answer:

$$\boxed{0.4\text{ m}^2}$$